Solvency requirements for life annuities allowing for the longevity risk: Internal models versus standard formulas

Ermanno Pitacco
Università di Trieste
ermanno.pitacco@econ.units.it
Presentation mostly based on research and teaching material, jointly with Annamaria Olivieri (University of Parma)
INTRODUCTION

The annuity provider takes, according to policy conditions (options and guarantees)

- financial risks
- biometric risks

Focus on biometric risks

Risk arising from *individual lifetimes* is a “process risk” (originated by random mortality fluctuations), called *individual longevity risk*, and can be diversified by increasing the portfolio size or via reinsurance arrangements, i.e. inside the traditional insurance-reinsurance process

Risk arising from *average lifetime in the population of annuitants* (originated by future unknown mortality level / trend) is a “systematic risk”, called *aggregate longevity risk*, and cannot be diversified inside the traditional insurance-reinsurance process
Main features of mortality trends

- Decreasing infant mortality
- Decreasing annual probabilities of dying, over wide age ranges, in particular at old ages
- Increasing life expectancy (both at the birth, and at old ages)
- Increasing modal duration of life (Lexis point)
- Changes in the shapes of survival functions:
  - rectangularization (but at old ages)
  - expansion
Mortality forecasts and uncertainty in future trend

Observed mortality trends $\Rightarrow$ construction of projected mortality tables, including a forecast of future mortality

Whatever projection adopted, future mortality trend is unknown $\Rightarrow$ systematic risk borne by the annuity provider

Capital allocation must face the systematic risk in particular
FROM SOLVENCY 1 TO SOLVENCY 2 IN LIFE INSURANCE

COMPARING THE APPROACHES

Solvency 1
Approach based on a simple short-cut formula

\[ M = 0.04 c_1 V + 0.003 c_2 (C - V)^{(+)} + A_1 + A_2 \]

Note that:

- investment risks quantified by the portfolio reserve \( V \), corresponding to assets backing the liabilities, hence disregarding assets backing the shareholders’ capital

- calculation of the reserve with a strong prudential basis leads to a higher margin requirement (!)

- mortality risks quantified by \( (C - V)^{(+)} \) ⇒ insurance products with negative sum at risk (e.g. life annuities) disregarded
From Solvency 1 to Solvency 2 . . . (cont’d)

- no parameter, other than \( c_1, c_2 \) (related to reinsurance), expresses the specific risk profile of an insurance company, for example:
  - types of investments and diversification
  - types of insurance products
  - policy conditions (e.g. option to annuitize)
  - . . .

- formula applicable to companies working in “homogeneous” insurance markets

**Solvency 2**

Approach based on Enterprise Risk Management (ERM) principles
- involving the whole insurance activity
- relying not only on quantitative requirements
THE ERM APPROACH

Focus on life annuities

According to a (rather) comprehensive point of view, a life annuity is

- for the individual: a tool for transferring risks to an annuity provider (an insurer, in particular)
- for the annuity provider: a risky position with prospect of profit

Amount of

- residual risk borne by the individual
- risk taken by the annuity provider

depends on the product design, namely on guarantees and options included in the life annuity product

ERM approach can provide guidelines in the analysis of risks, under both the individual perspective (a personal wealth management and RM problem) and the insurer’s perspective (an insurance RM problem)
The Risk-Management process

RM is a never-ending process

Phases in the RM process
In general

- *Risk identification*: what risks should be allowed for in our analysis?

- *Risk assessment*: what is the impact of risks on results (cash flows, profits, net asset value)?

- *Analysis of actions*: how to mitigate the impact of risks? how to fund possible losses?

- *Choice of actions*: define an appropriate mix, based on a cost vs benefit analysis

- *Monitoring*: what the effectiveness of actions undertaken in the (possibly innovated) scenario?
Analysis of actions. Choice of actions

Aim of this Section: define a framework for what follows

A wide range of actions available, aiming at risk mitigation

Actions:

- **Product design**
  - Guarantees and options
  - Pricing
  - Participation mechanisms

- **Portfolio protection**
  - Natural hedging
  - Risk transfer
  - Capital allocation
  - ...
From Solvency 1 to Solvency 2 . . . (cont’d)

Each action implies benefits and costs, as well as residual risk
For example:

- product design can make the product more or less attractive to potential policyholders, more or less risky to the insurer
- risk transfer to a reinsurer implies a cost to the cedant
- capital allocation implies a cost to the shareholders

Appropriate mix of actions should be adopted
Refer to a portfolio consisting of a generation of immediate life annuities
See following Figure
A range of actions available (at least in principle) in order to
- raise the threshold
- modify the cash flow profile (reduce, smooth, etc.)
Annual outflows in a life annuity portfolio (one generation)
From Solvency 1 to Solvency 2 . . . (cont’d)

Actions aiming at risk mitigation
In terms of RM actions:

Loss control

- **Loss prevention**
  - pricing: \( (1) \Rightarrow (a) \)
    - use of an appropriate projected table
    - premium calculation principle

- **Loss reduction**
  - pricing: \( (1) \Rightarrow (a) \)
  - profit participation: \( (4) \Rightarrow (b) \)
  - reduction of the annuity amount: \( (5) \Rightarrow (b) \)
Loss financing

- **Risk transfer**
  - traditional reinsurance arrangements: \((6) \Rightarrow (b)\)
    as regards the systematic component of longevity risk, traditional reinsurance does not provide by itself a solution, as the risk cannot be diversified by increasing (reinsurer’s) portfolio size
    traditional reinsurance can work provided that
    - the reinsurer experiences easier natural hedging
    - a further transfer (to capital markets) is feasible
  - swap-like reinsurance: \((7) \Rightarrow (b)\)
  - ART, viz longevity bonds: \((8) \Rightarrow (b)\)
• **Capital allocation**
  ◦ shareholders’ capital: \(2 \Rightarrow (a), \ (3) \Rightarrow (a)\)

• **Natural hedging** (not represented in figures)
  ◦ “across LOBs”: insurance products with a negative sum at risk (viz life annuities) & insurance products with a positive sum at risk (endowments, assurances)
  ◦ “across time”: life annuities with death benefit decreasing as the age at death increases
SAFE-SIDE RESERVE VERSUS BEST-ESTIMATE RESERVE

Traditional approach to reserving ⇒ adoption of the first-order basis in discounting future benefits and premiums

Hence, the (individual) reserve $V_t$ constitutes a prudential (or “safe-side”) evaluation of the insurer’s liability

“Degree” of prudence cannot be easily determined

Different approach to reserving ⇒ explicitly allowing for risks and for a chosen prudence target

Assume, for discounting future benefits and future premiums, the second-order (or realistic) basis

⇒ “best estimate” assessment of the policy reserve

⇒ best-estimate reserve (or central-estimate reserve) $V_t^{[BE]}$
Adequacy requirements in life insurance . . . (cont’d)

Difference $V_t - V_t^{[\text{BE}]} = \text{safety margin implied by the adoption of the first-order basis in the policy reserve } V_t$

In particular, adverse fluctuations in mortality / longevity can be faced thanks to this margin

**The risk margin**

Move from a single policy to a generation of “identical” policies

Assume that, at time $t$, the portfolio consists of $N_t$ policies

Then

- traditional safe-side reserve = $N_t V_t$
- best-estimate reserve = $N_t V_t^{[\text{BE}]}$

Safety margin, or *risk margin*, at the portfolio level given by:

$$RM_t = N_t (V_t - V_t^{[\text{BE}]})$$
Adequacy requirements in life insurance . . . (cont’d)

Sound approach to the management of the insurer’s risks:

- appropriate quantification of the relevant impact on portfolio results
- rather than starting from a generic prudential assessment of the reserve and then finding the resulting risk margin, the risk margin should be determined depending on the insurer’s risk profile, quantified by a convenient risk measure
Adequacy requirements in life insurance . . . (cont’d)

Consider the following quantities, referred to the residual portfolio duration:

- \( Y^{[P]}(t, m) \) = random present value at time \( t \) of future benefits
- \( X^{[P]}(t, m) \) = random present value at time \( t \) of future premiums
- \( Z^{[P]}(t, m) \) = random present value at time \( t \) of the portfolio result

Assume that an amount of assets equal to the best-estimate reserve is available; then:

\[
Z^{[P]}(t, m) = N_t V_t^{[BE]} + X^{[P]}(t, m) - Y^{[P]}(t, m)
\]

It is possible to prove that, according to the realistic basis, we have

\[
\mathbb{E}[Z^{[P]}(t, m)] = 0
\]
Adequacy requirements in life insurance . . . (cont’d)

If only the amount $N_t V_t^{[BE]}$ is available to meet future benefits net of future premiums ⇒ probability of a negative result is very high (see Figure left)

\[ \alpha \]

\[ \text{VaR}_\alpha \]

\[ \text{E}[Z^{[P]}(t,m)] \]

\[ \text{E}[Z^{[P][RM]}(t,m)] = RM_t \]

The prob. distribution of $Z^{[P]}(t,m)$

The prob. distribution of $Z^{[P][RM]}(t,m)$
Adequacy requirements in life insurance . . . (cont’d)

A further amount should be available, determined via appropriate risk measures

For example, take the VaR at a given confidence level \( 1 - \alpha \), and set:

\[
R M_t = -VaR_{\alpha}
\]

Assume that the amount \( R M_t \) “belongs” to the portfolio (financed, at least to some extent, by the safety loadings embedded in the premiums already cashed)

Random present value of the portfolio result redefined as follows:

\[
Z^{[P][RM]}(t, m) = Z^{[P]}(t, m) + RM_t = N_t V^{[BE]}_t + RM_t + X^{[P]}(t, m) - Y^{[P]}(t, m)
\]

(see Figure right)
**The Portfolio Liability and Beyond**

Assume

- the portfolio reserve, $V_t^{[P]}$, calculated as the best-estimate reserve plus the risk margin

- the risk margin given by the VaR at a stated confidence level; clearly, it depends on the portfolio size $N_t$:

$$V_t^{[P]} = N_t V_t^{[BE]} + RM_t(N_t); \quad t = 0, 1, 2, \ldots \quad (*)$$

Refer to a portfolio initially consisting of $N_0$ “identical” policies ($N_0$ given number)

At (future) time $t \Rightarrow$ the portfolio reserve is a random amount

For any given sequence $n_1, n_2, \ldots$ of numbers of policies in-force

$\Rightarrow$ estimated future portfolio reserve:

$$\hat{V}_t^{[P]} = n_t V_t^{[BE]} + RM_t(n_t) \quad (**)$$
Portfolio liabilities are counterbalanced by assets
If the assets have to be assessed at their market value (the “true” or “fair” value) ⇒ the related liabilities should be assessed, for consistency, at market value
The so-called *mark-to-market* approach to liability assessment should be adopted
A problem arises: is a (reliable) market value of liabilities available ?
Insurer’s liabilities are only traded in markets which cannot provide a reliable fair value (for example, the reinsurance market) ⇒ application of the mark-to-market approach restricted to liabilities which can be perfectly hedged by assets traded on appropriate markets (viz unit-linked insurance products)
Conversely, Eqs. (*) and (**) implement the *mark-to-model* approach to the assessment of the portfolio liabilities ⇒ based on actuarial models
Adequacy requirements in life insurance . . . (cont’d)

More assets than those just backing the fair value of the liabilities usually needed to face risks

Shareholders’ capital must be allocated and assigned to the portfolio

The amount to be allocated to a portfolio (and, more in general, to a life insurance business) determined according to a stated “solvency target”

Total amount of assets backing the insurer’s liabilities and shareholders’ capital must fulfill the adequacy requirement

1. as required by the supervisory authorities, quantified
   (a) “standard” formula
   (b) internal model

2. as stated by the company management, if this results in an amount higher than 1

Shareholders’ capital

- needed to fulfill the adequacy requirement ⇒ required capital
- (possibly) remaining amount ⇒ excess capital
Adequacy requirements in life insurance . . . (cont’d)

Assets, liabilities and shareholders’ capital (1)
Adequacy requirements in life insurance . . . (cont’d)

Risk Margin: The “Cost of Capital” Approach

Approach to the calculation of the risk margin ⇒ practicable alternative to the VaR approach

Define the target capital at time \( t \), \( TC_t \), as the amount of assets net of liabilities, required (in particular, by the supervisory authority) for solvency purposes

- assets assessed at their market value
- liabilities assessed at their best-estimate value

Assume that the target capital consists of two components

- the risk margin \( RM_t \)
- the solvency capital requirement \( SCR_t \)

(see Figure)

\[
TC_t = RM_t + SCR_t
\]
Adequacy requirements in life insurance . . . (cont’d)

Adequacy requirement fulfilled by:

1. the best-estimate reserve
2. the risk margin
3. the solvency capital requirement

Assets, liabilities and shareholders’ capital (2)
Adequacy requirements in life insurance . . . (cont’d)

Assume that \( SCR_t \) can be determined (at least approx) by adopting a given formula.

The risk margin \( RM_t \) is then defined as the cost of the solvency capital required for the run-off of the portfolio in the case of insurer’s default at the end of the current year.

Hence:

- the risk margin makes possible the run-off of the portfolio after default.
- without risk margin, no other insurer would be available to be charged with the portfolio itself.
- the risk margin “belongs” to the policyholders, because in the case of default it must be transferred together with the portfolio.
  \( \Rightarrow \) it is not a part of the shareholders’ capital.
Adequacy requirements in life insurance . . . (cont’d)

Refer to a portfolio of identical policies, total duration \( m \) years
\( \rho = \text{risk discount rate}; \rho > r_f, \text{where } r_f = \text{risk-free rate} \)
Assume that:
- \( \rho \) is the return required on the shareholders’ capital
- the capital allocated to the portfolio is invested at the risk-free rate
Spread \( \rho - r_f = \text{cost of capital not covered by the investment yield} \)
Risk margin at time \( t \) formally defined as follows:

\[
RM_t = (\rho - r_f) \left( \widehat{SCR}_{t+1} (1 + r_f)^{-1} + \widehat{SCR}_{t+2} (1 + r_f)^{-2} + \cdots + \widehat{SCR}_{m-1} (1 + r_f)^{-m+t+1} \right)
\]

where \( \widehat{SCR}_{t+h} = \text{estimate, at time } t, \text{ of the solvency capital requirement at time } t + h \)
THE LONGEVITY RISK: INTERNAL MODELS

See [Olivieri and Pitacco [2009b]]

**NOTATION AND ASSUMPTIONS**

- $T_{x\tau} = \text{remaining lifetime of an individual current age } x, \text{ born in calendar year } \tau$
- $H(\tau) = $ assumption about the future age pattern of mortality for cohort $\tau$
- $\mathcal{H}(\tau) = \text{set of alternative mortality assumptions for cohort } \tau$
- $\mathcal{H}(\tau) = \{H_1(\tau), H_2(\tau), \ldots, H_m(\tau)\}$
- $\rho_h = \text{weight assigned to assumption } H_h(\tau), \text{ with } 0 \leq \rho_h \leq 1 \text{ for } h = 1, 2, \ldots, m \text{ and } \sum_{h=1}^{m} \rho_h = 1$
The longevity risk: internal models (cont’d)

- $q_{x\tau}$ = one-year probability of dying
- Heligman-Pollard assumption (3rd term)

\[
\frac{q[H(\tau)]}{1 - q[H(\tau)]} = G[H(\tau)] \left( K[H(\tau)] \right)^x
\]

|               | $G[H_h(1955)]$ | $K[H_h(1955)]$ | $E[T_{65,1955}|H_h(1955)]$ |
|---------------|-----------------|-----------------|-----------------------------|
| $H_1(1955)$   | 5.561E-07       | 1.153           | 19.874                      |
| $H_2(1955)$   | 9.542E-07       | 1.144           | 20.442                      |
| $H_3(1955)$   | 1.414E-06       | 1.137           | 21.099                      |
| $H_4(1955)$   | 2.005E-06       | 1.130           | 21.849                      |
| $H_5(1955)$   | 2.533E-06       | 1.125           | 22.711                      |
| $H_6(1955)$   | 2.972E-06       | 1.121           | 23.676                      |
| $H_7(1955)$   | 3.294E-06       | 1.118           | 24.706                      |

*Table 1 - Parameters for the Heligman-Pollard law*
The longevity risk: internal models (cont’d)

Survival functions

Curves of deaths
The longevity risk: internal models (cont’d)

- $N_t =$ random number of annuitants at time $t$, $t = 0, 1, \ldots, \omega - x_0$
- $\Pi_t = \{ j \mid T_{x_0}^{(j)} \tau > t \} =$ in-force portfolio at time $t$
- $B_t^{(\Pi)} = b \ N_t =$ annual outflows for the portfolio, for $t = 1, 2, \ldots, \omega - x_0$
- random present value of future portfolio outflows:

$$Y_t^{(\Pi)} = \sum_{h=t+1}^{\omega-x_0} B_{h}^{(\Pi)} (1 + i)^{-(h-t)} = \sum_{h=t+1}^{\omega-x_0} b \ N_h (1 + i)^{-(h-t)}$$

- $A_t =$ amount of portfolio assets at time $t$

$$A_t = A_{t-1} (1 + i) - b \ N_t$$
The longevity risk: internal models (cont’d)

- $V_t^{(\Pi)} = \text{portfolio reserve (traditionally according to a prudential basis)}$
- $V_t^{(\Pi)[BE]} = \text{best-estimate reserve portfolio reserve}$
- $RM_t = \text{risk margin}$
- $V_t^{(\Pi)[BE]} + RM_t = \text{portfolio reserve as the market value of liabilities}$
- $M_t = A_t - V_t^{(\Pi)[BE]} = \text{assets available to meet the risks, backing both}$
  - the risk margin (included into the portfolio reserve)
  - the required capital
The longevity risk: internal models (cont’d)

**CAPITAL REQUIREMENTS ACCORDING TO INTERNAL MODELS**

Rules for assessing the capital required at time $z$

- **[R1]**: $\mathbb{P} [(M_{z+1} \geq 0) \land (M_{z+2} \geq 0) \land \cdots \land (M_{z+T} \geq 0)] = 1 - \varepsilon_1$
- **[R2]**: $\mathbb{P} [M_{z+T} \geq 0] = 1 - \varepsilon_2$
- **[R3]**: $\mathbb{P} \left[ (A_{z+1} - Y_{z+1}^{(\Pi)} \geq 0) \land (A_{z+2} - Y_{z+2}^{(\Pi)} \geq 0) \land \cdots \land (A_{z+T} - Y_{z+T}^{(\Pi)} \geq 0) \right] = 1 - \varepsilon_3$

where

- $\varepsilon_i = $ accepted default probability under rule $R_i \ (i = 1, 2, 3)$
- $T = $ time horizon
The longevity risk: internal models (cont’d)

CAPITAL REQUIREMENTS ACCORDING TO SOLVENCY 2

Capital charge for longevity risk at time $z$:

$$\text{Life}_{\text{long},z} = \Delta \text{NAV} | \text{longevity shock}$$

(longevity shock defined as a (permanent) decrease in the mortality rate at each age, in respect of the best-estimate assumption) with

$$\text{NAV}_z = A_z - V_z^{(\Pi)[BE]}$$

For example:

$$\Delta \text{NAV}_z = \text{Life}_{\text{long},z} = V_z^{(\Pi)[-25\%]} - V_z^{(\Pi)[BE]}$$

Risk margin defined according to a Cost-of-Capital logic
The longevity risk: internal models (cont’d)

Assets backing $\text{Life}_{\text{long},z}$ + assets backing $RM_z$ represent, in total, the amount of money facing risks.

To compare results obtained with internal models and within the Solvency 2 framework, define:

$$M_z^{[\text{Solv2}]} = \text{Life}_{\text{long},z} + RM_z$$

as the total amount of money required, under Solvency 2, to face longevity risks.
NUMERICAL EXAMPLES

Refer to a portfolio of immediate annuities

- one cohort of annuitants, initial age $x_0 = 65$
- individual annual benefit $b = 1$ in arrears
- lifetimes assumed independent and identically distributed
- $i = 0.03$

Mortality assumptions

- see Table 1
- best-estimate assumption $H_4(1955)$
- probability distribution over the set of mortality assumptions

$$\rho_1 = \rho_7 = 0.05; \quad \rho_2 = \rho_3 = \rho_5 = \rho_6 = 0.1; \quad \rho_4 = 0.5$$
Numerical examples (cont’d)

**Table 2**: Individual reserve net and inclusive of the risk margin required by Solvency 2, respectively $V_{z}^{(1)[BE]}$ and $V_{z}^{(1)}$

*Remark*

These quantities are defined as (or based on) expected values $\Rightarrow$ if $n_{z}$ is the number of annuitants at time $z$, then $V_{z}^{(\Pi)[BE]} = n_{z} V_{z}^{(1)[BE]}$ and $V_{z}^{(\Pi)} = n_{z} V_{z}^{(1)}$

<table>
<thead>
<tr>
<th>time $z$</th>
<th>$V_{z}^{(1)[BE]}$</th>
<th>$V_{z}^{(1)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15.259</td>
<td>16.342</td>
</tr>
<tr>
<td>5</td>
<td>12.956</td>
<td>13.887</td>
</tr>
<tr>
<td>10</td>
<td>10.599</td>
<td>11.357</td>
</tr>
<tr>
<td>15</td>
<td>8.294</td>
<td>8.874</td>
</tr>
<tr>
<td>20</td>
<td>6.167</td>
<td>6.581</td>
</tr>
<tr>
<td>25</td>
<td>4.336</td>
<td>4.611</td>
</tr>
<tr>
<td>30</td>
<td>2.877</td>
<td>3.050</td>
</tr>
<tr>
<td>35</td>
<td>1.807</td>
<td>1.913</td>
</tr>
</tbody>
</table>

Table 2 - Individual reserve
Numerical examples (cont’d)

Table 3: Amount of capital (per unit of portfolio reserve net of the risk margin, $V_{z}^{(II)(BE)}$) required according to the solvency rules [$R1$] and [$R3$] for several portfolio sizes

Rule [$R1$]: maximum possible time-horizon has been chosen, i.e. the time to the portfolio exhaustion, $T = \omega + 1 - x_0 - z$

The two requirements lead to similar outputs, at least when mortality only is addressed ⇒ the outputs suggest that rule [$R1$] is to some extent independent of the reserve when $T$ takes the maximum possible value for the time-horizon. Indeed, between random fluctuations and systematic deviations in longevity, the latter have the most severe impact

The aggregate longevity risk is long-term ⇒ to some extent the total related loss is more important than the timing of its emergence

Note that $M_{z}^{[R1]}(T)$ and $M_{z}^{[R3]}$ represent the assets required to meet the risks ⇒ the amount must partially back the risk margin, to be included into the technical provision, and the required capital
Numerical examples (cont’d)

<table>
<thead>
<tr>
<th>time $z$</th>
<th>$n_0 = 100$</th>
<th>$n_0 = 1000$</th>
<th>$n_0 = 10000$</th>
<th>$n_0 = 100$</th>
<th>$n_0 = 1000$</th>
<th>$n_0 = 10000$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>13.647%</td>
<td>10.730%</td>
<td>9.819%</td>
<td>13.647%</td>
<td>10.730%</td>
<td>9.819%</td>
</tr>
<tr>
<td>5</td>
<td>17.588%</td>
<td>13.870%</td>
<td>12.757%</td>
<td>17.573%</td>
<td>13.868%</td>
<td>12.757%</td>
</tr>
<tr>
<td>10</td>
<td>22.915%</td>
<td>18.240%</td>
<td>16.813%</td>
<td>22.880%</td>
<td>18.240%</td>
<td>16.813%</td>
</tr>
<tr>
<td>15</td>
<td>29.927%</td>
<td>23.757%</td>
<td>22.125%</td>
<td>29.759%</td>
<td>23.753%</td>
<td>22.125%</td>
</tr>
<tr>
<td>20</td>
<td>41.014%</td>
<td>31.319%</td>
<td>29.205%</td>
<td>40.742%</td>
<td>31.308%</td>
<td>29.205%</td>
</tr>
<tr>
<td>25</td>
<td>56.901%</td>
<td>42.052%</td>
<td>38.545%</td>
<td>56.472%</td>
<td>42.025%</td>
<td>38.545%</td>
</tr>
<tr>
<td>30</td>
<td>86.748%</td>
<td>57.285%</td>
<td>50.622%</td>
<td>84.647%</td>
<td>57.210%</td>
<td>50.605%</td>
</tr>
<tr>
<td>35</td>
<td>180.337%</td>
<td>86.445%</td>
<td>66.553%</td>
<td>170.443%</td>
<td>85.749%</td>
<td>66.450%</td>
</tr>
</tbody>
</table>

Table 3 - Required assets to meet risks based on rules $[R1]$ and $[R3]$, facing individual and aggregate longevity risk.
Table 4: Outputs from rule $[R1]$ are investigated for shorter time-horizons.

Comparing Table 3 with Table 4 $\Rightarrow$ the long-term nature of aggregate longevity risk clearly emerges.

Note that, both in Table 3 and Table 4, at each valuation time and for each rule, the amount of required assets decreases when a larger portfolio is considered, obviously thanks to the pooling nature of random fluctuations (individual longevity risk).
### Numerical examples (cont’d)

<table>
<thead>
<tr>
<th>time $z$</th>
<th>time-horizon $T = 1$</th>
<th>time-horizon $T = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n_0 = 100$</td>
<td>$n_0 = 1,000$</td>
</tr>
<tr>
<td>0</td>
<td>0.574%</td>
<td>0.473%</td>
</tr>
<tr>
<td>5</td>
<td>1.058%</td>
<td>0.743%</td>
</tr>
<tr>
<td>10</td>
<td>1.951%</td>
<td>1.159%</td>
</tr>
<tr>
<td>15</td>
<td>3.600%</td>
<td>2.034%</td>
</tr>
<tr>
<td>20</td>
<td>6.639%</td>
<td>3.602%</td>
</tr>
<tr>
<td>25</td>
<td>12.246%</td>
<td>6.338%</td>
</tr>
<tr>
<td>30</td>
<td>22.588%</td>
<td>12.168%</td>
</tr>
<tr>
<td>35</td>
<td>41.664%</td>
<td>26.210%</td>
</tr>
</tbody>
</table>

**Table 4** - Required assets to meet risks based on rule $[R1]$, per unit of portfolio reserve (net of the risk margin): $M_z^{[R1](T)}(T) / V_z^{(II)[BE]}$, facing individual and aggregate longevity risks
Table 5: Results address random fluctuations only (individual longevity risk). Required amount of assets calculated adopting the best-estimate mortality assumption $H_4(1955)$

Table 6: Only aggregate longevity risk has been accounted for, by assuming that whatever is the realized mortality trend, the actual number of deaths in each year is the same as expected under the relevant trend assumption. Required amount of assets per unit of portfolio reserve (net of the risk margin) independent of the size of the portfolio, due to the systematic nature of aggregate longevity risk

Tables 5 & 6: Some useful information ⇒ implementing an internal model allowing for a component only of a risk represents an improper use of the model itself

Note that on summing up the results in Table 5 and Table 6, for a given rule and portfolio size, we do not find the correspondent results in Table 3 or Table 4 ⇒ some aspects missed when working with marginal distributions only
**Numerical examples (cont’d)**

### Table 5 - Required assets to meet risks based on rules [R1] and [R3],

*face individual longevity risk only; mortality assumption $H_4(1955)$*

<table>
<thead>
<tr>
<th>time $z$</th>
<th>$n_0 = 100$</th>
<th>$n_0 = 1 000$</th>
<th>$n_0 = 10 000$</th>
<th>$n_0 = 100$</th>
<th>$n_0 = 1 000$</th>
<th>$n_0 = 10 000$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7.813%</td>
<td>2.832%</td>
<td>0.879%</td>
<td>7.031%</td>
<td>2.698%</td>
<td>0.800%</td>
</tr>
<tr>
<td>5</td>
<td>9.983%</td>
<td>3.071%</td>
<td>1.067%</td>
<td>9.436%</td>
<td>2.949%</td>
<td>1.040%</td>
</tr>
<tr>
<td>10</td>
<td>12.144%</td>
<td>4.040%</td>
<td>1.217%</td>
<td>11.543%</td>
<td>3.759%</td>
<td>1.193%</td>
</tr>
<tr>
<td>15</td>
<td>16.153%</td>
<td>5.202%</td>
<td>1.544%</td>
<td>14.982%</td>
<td>4.921%</td>
<td>1.462%</td>
</tr>
<tr>
<td>20</td>
<td>22.343%</td>
<td>6.938%</td>
<td>2.091%</td>
<td>21.292%</td>
<td>6.554%</td>
<td>1.936%</td>
</tr>
<tr>
<td>25</td>
<td>29.728%</td>
<td>10.388%</td>
<td>3.072%</td>
<td>28.546%</td>
<td>9.642%</td>
<td>2.983%</td>
</tr>
<tr>
<td>30</td>
<td>54.183%</td>
<td>16.871%</td>
<td>5.547%</td>
<td>51.253%</td>
<td>16.807%</td>
<td>5.152%</td>
</tr>
<tr>
<td>35</td>
<td>155.859%</td>
<td>36.795%</td>
<td>11.715%</td>
<td>144.058%</td>
<td>34.809%</td>
<td>11.207%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>time $z$</th>
<th>$n_0 = 100$</th>
<th>$n_0 = 1 000$</th>
<th>$n_0 = 10 000$</th>
<th>$n_0 = 100$</th>
<th>$n_0 = 1 000$</th>
<th>$n_0 = 10 000$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.574%</td>
<td>0.473%</td>
<td>0.171%</td>
<td>1.834%</td>
<td>0.983%</td>
<td>0.378%</td>
</tr>
<tr>
<td>5</td>
<td>1.058%</td>
<td>0.743%</td>
<td>0.271%</td>
<td>3.358%</td>
<td>1.443%</td>
<td>0.479%</td>
</tr>
<tr>
<td>10</td>
<td>1.951%</td>
<td>0.932%</td>
<td>0.388%</td>
<td>5.162%</td>
<td>1.957%</td>
<td>0.657%</td>
</tr>
<tr>
<td>15</td>
<td>3.600%</td>
<td>1.642%</td>
<td>0.583%</td>
<td>8.604%</td>
<td>2.630%</td>
<td>0.932%</td>
</tr>
<tr>
<td>20</td>
<td>6.639%</td>
<td>2.458%</td>
<td>0.806%</td>
<td>13.304%</td>
<td>3.775%</td>
<td>1.329%</td>
</tr>
<tr>
<td>25</td>
<td>12.246%</td>
<td>4.633%</td>
<td>1.379%</td>
<td>19.609%</td>
<td>7.129%</td>
<td>2.166%</td>
</tr>
<tr>
<td>30</td>
<td>22.588%</td>
<td>7.878%</td>
<td>2.804%</td>
<td>41.023%</td>
<td>13.181%</td>
<td>4.168%</td>
</tr>
<tr>
<td>35</td>
<td>41.664%</td>
<td>21.058%</td>
<td>7.321%</td>
<td>124.167%</td>
<td>32.954%</td>
<td>10.176%</td>
</tr>
</tbody>
</table>
### Numerical examples (cont’d)

<table>
<thead>
<tr>
<th>time $z$</th>
<th>$\frac{M_z^{[R1]}(1)}{V_z^{(III)[BE]}}$</th>
<th>$\frac{M_z^{[R1]}(3)}{V_z^{(III)[BE]}}$</th>
<th>$\frac{M_z^{[R1]}(\omega + 1 - x_0 - z)}{V_z^{(III)[BE]}}$</th>
<th>$\frac{M_z^{[R3]}}{V_z^{(III)[BE]}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.110%</td>
<td>0.366%</td>
<td>9.402%</td>
<td>9.402%</td>
</tr>
<tr>
<td>5</td>
<td>0.247%</td>
<td>0.805%</td>
<td>12.270%</td>
<td>12.270%</td>
</tr>
<tr>
<td>10</td>
<td>0.531%</td>
<td>1.691%</td>
<td>16.134%</td>
<td>16.134%</td>
</tr>
<tr>
<td>15</td>
<td>1.105%</td>
<td>3.423%</td>
<td>21.301%</td>
<td>21.301%</td>
</tr>
<tr>
<td>20</td>
<td>2.233%</td>
<td>6.681%</td>
<td>28.102%</td>
<td>28.102%</td>
</tr>
<tr>
<td>25</td>
<td>4.389%</td>
<td>12.511%</td>
<td>36.826%</td>
<td>36.826%</td>
</tr>
<tr>
<td>30</td>
<td>8.346%</td>
<td>22.240%</td>
<td>47.599%</td>
<td>47.599%</td>
</tr>
<tr>
<td>35</td>
<td>15.214%</td>
<td>36.932%</td>
<td>60.269%</td>
<td>60.269%</td>
</tr>
</tbody>
</table>

**Table 6** - Required assets to meet risks based on rules $[R1]$ and $[R3]$, facing aggregate longevity risk only
Table 7: Amount of assets required under Solvency 2 to meet risks, split into risk margin (to be included into the technical provision) and required capital. Amount independent of the portfolio size

No specific capital allocation is required for the risk of random fluctuations, treated as hedgeable risk
Numerical examples  (cont’d)

<table>
<thead>
<tr>
<th>time $z$</th>
<th>$\frac{M_{\text{Solv}^2}[z]}{V_z[\Pi][BE]}$</th>
<th>$\frac{RM_z}{V_z[\Pi][BE]}$</th>
<th>$\frac{\text{Life}_{\text{long},z}}{V_z[\Pi][BE]}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>14.369%</td>
<td>7.095%</td>
<td>7.274%</td>
</tr>
<tr>
<td>5</td>
<td>16.266%</td>
<td>7.186%</td>
<td>9.080%</td>
</tr>
<tr>
<td>10</td>
<td>18.533%</td>
<td>7.156%</td>
<td>11.377%</td>
</tr>
<tr>
<td>15</td>
<td>21.288%</td>
<td>6.995%</td>
<td>14.293%</td>
</tr>
<tr>
<td>20</td>
<td>24.713%</td>
<td>6.712%</td>
<td>18.000%</td>
</tr>
<tr>
<td>25</td>
<td>29.123%</td>
<td>6.356%</td>
<td>22.767%</td>
</tr>
<tr>
<td>30</td>
<td>35.123%</td>
<td>6.020%</td>
<td>29.102%</td>
</tr>
<tr>
<td>35</td>
<td>43.922%</td>
<td>5.856%</td>
<td>38.067%</td>
</tr>
</tbody>
</table>

*Table 7 - Required assets to meet risks according to Solvency 2*
Numerical examples (cont’d)

**Tables 6 & 7:** Required capital flat in respect of the portfolio size (at any valuation time) suggests a deterministic approach for allocating capital to deal with aggregate longevity risk. In particular, assessment of required capital could be based on a comparison between the actual reserve (net of the risk margin) and a reserve (again net of the risk margin) calculated under a more severe longevity trend assumption (as in Solvency 2).

Let $V_z^{(\Pi)[B]}$ = expected present value of future payments for the portfolio, based on a longevity assumption higher than the best-estimate $V_z^{(\Pi)[BE]}$. Let $V_z^{(\Pi)[BE]} \leq V_z^{(\Pi)[B]}$.

$V_z^{(\Pi)[B]}$ = technical provision, net of the risk margin, based on assumption $B$.

Required amount of assets to meet risks then:

$$[R4] : M_z^{[R4]} = V_z^{(\Pi)[B]} - V_z^{(\Pi)[BE]}$$
Numerical examples (cont’d)

Note that \([R4]\)

- accounts for aggregate longevity risk only
- no default probability is explicitly considered
- time-horizon implicitly considered = the maximum residual duration of the portfolio, given that this is the time-horizon referred to in the calculation of the reserve
- \(M^{[R4]}_z\) is a difference between expected values \(\Rightarrow\) linear in respect of the portfolio size

To compare rules \([R1]\), \([R3]\) and \([R4]\), define the following ratios:

\[
QM^{[R1]}_z(T; n_z) = \frac{M^{[R1]}_z(T)}{V^{(\Pi)(BE)}_z}; \quad QM^{[R3]}_z(n_z) = \frac{M^{[R3]}_z}{V^{(\Pi)(BE)}_z}
\]

\[
QV_z = \frac{M^{[R4]}_z}{V^{(\Pi)(BE)}_z}
\]
Numerical examples (cont’d)

Ratios $QM_z^{[R1]}(T; n_z)$ and $QM_z^{[R3]}(n_z)$ depend on the size of the portfolio as they also account for the risk of random fluctuations. Ratio $QV_z$ addresses just the aggregate longevity risk $\Rightarrow$ independent of the portfolio size.

On the other hand, $[R1]$ and $[R3]$ could be implemented considering only either the risk of random fluctuations or the risk of systematic deviations. When addressing the aggregate longevity risk only, ratios $QM_z^{[R1]}(T; n_z)$ and $QM_z^{[R3]}(n_z)$ are independent of the size of the portfolio $\Rightarrow$ suggests a deterministic approach instead of a simulation-based procedure.

However, addressing just a component of the mortality risks represents an improper use of rules $[R1]$ and $[R3]$.
Numerical examples (cont’d)

Further difference between ratios \( QM_z^{[R1]}(T;n_z) \) and \( QV_z \): possibility to set a preferred time-horizon; time-horizons other than the maximum one may be chosen only when rule \([R1]\) is adopted

Not possible to derive general conclusions regarding the comparison between the outcomes of levels of ratios \( QM_z^{[R1]}(T;n_z) \) and \( QM_z^{[R3]}(n_z) \), on one hand, and \( QV_z \), on the other

Note that the quantity \( \frac{\text{Lifelong}_{z}}{V_z^{(11)[BE]}} \) provides an example of ratio \( QV_z \)

A simplified rule, such as \([R4]\) and that implicit in Solvency 2, while more direct to implement, may result in an amount of required assets too high or too low in relation to the portfolio size and the duration of the portfolio, due to neglecting the pooling component, whose impact depend on portfolio size (and age as well)
Is the (immediate) life annuity product “too demanding”?
Do retirees consider the life annuity product “too risky”, and hence not attractive?

Possible alternative products
  - providing a post-retirement income
  - less exposed to (some of the) biometric risks

**Life annuity with a guarantee period**
The annual benefit is paid for the guarantee period (5 or 10 years, say) regardless of whether the annuitant is alive or not

For a guarantee period of $r$ years, single premium:

$$\Pi = b \, a'_{r\vert} + b \, r \, a'_{x}$$

with $a'_{r\vert} = $ present value of a temporary annuity-certain (interest rate $i'$)
Capital protection

An interesting feature of some life annuity products (value-protected life annuities)

An example of counter-insurance

In the case of early death of the annuitant, the difference (if positive) between the single premium and the cumulated benefits paid to the annuitant is paid to a predesignated beneficiary.

Usually, capital protection expires at some given age (75, say), after which nothing is paid even though the difference above is positive.

Possible hedging mechanism

Higher premium (depending on the extension of the “protected period”)
Annuitization strategies: delayed annuitization

Assume that, at time of retirement (age $y$)

- amount $S$ available to the retiree (result of accumulation process)
- the retiree can choose between two alternatives:
  1. to purchase an immediate life annuity, with annual benefit $b$ (annuitize amount $S$); see Figure, upper panel
  2. to leave amount $S$ in a fund, and then
     (a) withdraw the amount $b^{(1)}$ at times $h = 1, 2, \ldots, k$ (say, with $k = 5$ or $k = 10$) ⇒ temporary withdrawal process
     (b) convert at time $k$ the remaining amount $R$ into an immediate life annuity with annual benefit $b^{(2)}$ ⇒ delayed annuitization (provided she / he is alive)

see Figure, lower panel
Immediate annuitization versus delayed annuitization
Consider alternative 2

Amount $R$ available at time $k$ to buy the life annuity depends on the annual withdrawal $b^{(1)}$ and the interest rate $g$ credited to the non-annuitized fund

If $g = i$ (the interest rate in the pricing basis of the life annuity) and $b^{(1)} = b$ ⇒ amount $R$ not sufficient to purchase a life annuity with $b^{(2)} = b$, because of the absence of mutuality during the withdrawal period

Absence of mutuality can be compensated (at least in principle) by a higher investment yield, namely if $g > i$
Point of view of the retiree:

- advantages of delay in the purchase of the life annuity:
  - in the case of death before time \( k \), the fund available constitutes a bequest
  - more flexibility gained, as the annuitant may change her / his income profile modifying the withdrawal sequence (however, with possible change in the fund available at time \( k \))

- disadvantages:
  - risk of a shift to a different life table in the pricing basis
    \( \Rightarrow \) conversion rate at time \( k \) possibly less favorable to the annuitant
  - if \( k \) is high, difficult to gain the required yield avoiding too risky investments
CONCLUSIONS

Solvency 2

⇒ rethinking the insurance (- reinsurance) process
⇒ a new scope for life insurance technique (and for actuaries)

Traditional life insurance technique mainly focussed on products, and related pricing and reserving issues

Extending the range of topics requires a larger set of (mathematical, financial, ...) tools, and an appropriate approach

A (quantitative) RM perspective, implying the adoption of stochastic models, can provide an appropriate approach
Why is RM in insurance (and life insurance in particular) a recent achievement?

- insurance activities involve, by nature, the management of risks
- old traditional actuarial setting disregarded a rigorous risk-management approach
- awareness of risks $\Rightarrow$ underwriting process, reinsurance, capital allocation
- strength of traditional actuarial ideas (e.g. equivalence principle)
  $\Rightarrow$ actuarial techniques “self-contained”
  $\Rightarrow$ historical closure in respect of other disciplines
- new scenarios, new insurance products, new regulatory framework $\Rightarrow$ a sound risk-oriented approach
Some references


